

Name: _____

Instructor: _____

Math 10560, Practice Exam 1.

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.
- Each multiple choice question is worth 7 points. Your score will be the sum of the best 10 scores on the multiple choice questions plus your score on questions 13-16.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!					
1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
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11.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice _____

13. _____

14. _____

15. _____

Total _____

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Multiple Choice

1.(7 pts.) The function $f(x) = x^3 + 3x + e^{2x}$ is one-to-one. Compute $(f^{-1})'(1)$.

- (a) $\frac{1}{6+e}$ (b) $\frac{1}{6+2e}$ (c) 0 (d) $\frac{1}{4}$ (e) $\frac{1}{5}$

2.(7 pts.) Solve the following equation for x :

$$\ln(x+4) - \ln x = 1 .$$

- (a) There is no solution. (b) $x = \frac{4}{e-1}$
- (c) $x = \frac{4}{1-e}$ (d) $x = e+2$ and $x = e-2$
- (e) $x = \frac{4}{e-1}$ and $x = \frac{4}{e+1}$

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3.(7 pts.) Differentiate the function

$$f(x) = \frac{(x^2 - 1)^4}{\sqrt{x^2 + 1}}.$$

(a) $f'(x) = \frac{x(x^2 - 1)^4}{\sqrt{x^2 + 1}} \left(\frac{8}{x^2 - 1} + \frac{1}{x^2 + 1} \right)$

(b) $f'(x) = \frac{(x^2 - 1)^4}{\sqrt{x^2 + 1}} \left(\frac{4}{x^2 - 1} - \frac{1}{x^2 + 1} \right)$

(c) $f'(x) = \frac{(x^2 - 1)^4}{\sqrt{x^2 + 1}} \left(\frac{4}{x^2 - 1} + \frac{1}{x^2 + 1} \right)$

(d) $f'(x) = \frac{(x^2 - 1)^4}{\sqrt{x^2 + 1}} \left(\frac{8}{x^2 - 1} - \frac{1}{x^2 + 1} \right)$

(e) $f'(x) = \frac{x(x^2 - 1)^4}{\sqrt{x^2 + 1}} \left(\frac{8}{x^2 - 1} - \frac{1}{x^2 + 1} \right)$

4.(7 pts.) Compute the integral

$$\int_{2e}^{2e^2} \frac{1}{x \left(\ln \frac{x}{2} \right)^2} dx.$$

- (a) 2 (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) 1 (e) 0

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5.(7 pts.) Compute the limit

$$\lim_{x \rightarrow \infty} \frac{e^x - e^{-x}}{e^{2x} - e^{-2x}}.$$

- (a) 1 (b) ∞ (c) e
(d) 0 (e) does not exist

6.(7 pts.) Find $f'(x)$ if

$$f(x) = x^{\ln x}.$$

- (a) $2(\ln x)x^{\ln x}$
(b) $x^{\ln x} \ln x$
(c) $2(\ln x)x^{(\ln x)-1}$
(d) $x^{\ln x}(\ln x + 1)$
(e) $x^{(\ln x)-1} \ln x$

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7.(7 pts.) Calculate the following integral.

$$\int_0^1 \frac{\arctan x}{1+x^2} dx .$$

- (a) $\frac{1}{2}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi^2}{32}$ (d) $\ln 2$ (e) $\frac{\pi^2}{8}$

8.(7 pts.) If 100 grams of radioactive material with a half-life of two days are present at day zero, how many grams are left at day three?

- (a) $\frac{100}{4^{1/3}}$ (b) $\frac{100}{2^{1/3}}$ (c) $\frac{100}{\sqrt{2}}$ (d) $\frac{100}{\sqrt{8}}$ (e) 50

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9.(7 pts.) $\lim_{x \rightarrow 0^+} (\cos x)^{\frac{1}{x^2}} =$

- (a) e (b) $e^{-\frac{1}{2}}$ (c) ∞
(d) 1 (e) Does not exist

10.(7 pts.) The integral

$$\int_0^{\pi/2} x \cos(x) dx$$

is

- (a) $\frac{\pi}{2} - 1$ (b) divergent (c) 0
(d) $\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}$ (e) $1 - \frac{\pi}{2}$

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11.(7 pts.) Evaluate the integral

$$\int_0^{\pi/2} \sin^3(x) \cos^5(x) dx.$$

- (a) $\frac{1}{4}$ (b) $\frac{1}{24}$ (c) 0 (d) $-\frac{1}{24}$ (e) $\frac{\pi}{2}$

12.(7 pts.) If you expand $\frac{1+x-x^2}{x(x+1)^2}$ as a partial fraction, which expression below would you get?

- (a) $\frac{1}{x} - \frac{2}{x+1} + \frac{1}{(x+1)^2}$ (b) $\frac{-2}{x} + \frac{1}{x+1}$
(c) $\frac{1}{x} + \frac{1}{(x+1)^2}$ (d) $\frac{1}{x} - \frac{3}{(x+1)^2}$
(e) $\frac{1}{x} + \frac{1}{x+1} + \frac{1}{(x+1)^2}$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(12 pts.)

Evaluate

$$\int \frac{x^2}{\sqrt{9-x^2}} dx.$$

Note: The formula sheet will help you with this problem.

Write your answer in terms of the original variable x and (if needed) replace all composite trigonometric functions (such as $\cos(\sin^{-1}(x/n))$ etc...) by algebraic combinations of x .

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14. (15 pts.) Let $C(t)$ be the concentration of a drug in the bloodstream. As the body eliminates the drug, $C(t)$ decreases at a rate that is proportional to the amount of the drug that is present at the time. Thus $C'(t) = kC(t)$, where k is a constant. The initial concentration of the drug is 4 mg/ml. After 5 hours, the concentration is 3 mg/ml.

(a) Give a formula for the concentration of the drug at time t .

(b) How much drug will there be in 10 hours?

(c) How long will it take for the concentration to drop to 0.5 mg/ml?

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15. (3 pts.) Please circle “TRUE” if you think the statement is true, and circle “FALSE” if you think the statement is False.

(a)(1 pt. No Partial credit) $\int \frac{1}{1+x^2} = \ln|1+x^2| + C.$

TRUE FALSE

(b)(1 pt. No Partial credit) $2^x = e^{x \ln(2)}.$

TRUE FALSE

(c)(1 pt. No Partial credit) $\ln\left(\frac{1}{x}\right) = -\ln(x)$ for all $x > 0.$

TRUE FALSE

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The following is the list of useful trigonometric formulas:

Note: $\sin^{-1} x$ and $\arcsin(x)$ are different names for the same function and $\tan^{-1} x$ and $\arctan(x)$ are different names for the same function.

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}(\sin(x - y) + \sin(x + y))$$

$$\sin x \sin y = \frac{1}{2}(\cos(x - y) - \cos(x + y))$$

$$\cos x \cos y = \frac{1}{2}(\cos(x - y) + \cos(x + y))$$

$$\int \sec \theta = \ln |\sec \theta + \tan \theta| + C$$

$$\int \csc \theta = \ln |\csc \theta - \cot \theta| + C$$

$$\csc \theta = \frac{1}{\sin \theta}, \quad \cot \theta = \frac{1}{\tan \theta}$$

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